Question 1 **13 Marks** **Marks**

(a) Find the primitive of $(3x-2)^{10}$ **2**

(b) The gradient of a curve is given by $\frac{dy}{dx}=2x-3$. What is the equation of the curve if it passes through $(1,5)$ **2**

(c) The area enclosed by the curve $y=3x-x^2$, the x-axis, the lines $x=1$ and $x=2$, is rotated about the x-axis. Find the volume of the solid formed. Give your answer in terms of π. **4**

(d) In what ratio does the x axis divide the area of the region bounded by the parabolas $y=3x-x^2$ and $y=x^2-x$? **5**

Question 2 **14 Marks** ***Start a new page***

(a) Solve $\log_e(4-3x)=2\log_e x$. **4**

(b) Differentiate

(i) $\log_e(3-2x^2)$ **1**

(ii) $2x\ln 3x$ **2**

(c) If $y=x\log_e x$, find:

(i) $\frac{dy}{dx}$. **1**

(ii) Find the coordinates of the stationary point. **2**

(iii) Determine the nature of the stationary point. **1**

(iv) Find the value of y (correct to three decimal places) when $x=0.001$. **1**

(v) Use the information found to sketch the curve $y=x\log x$ **2**

Q3. … p.2

<u>Question 3</u> **<u>13 Marks</u>** ***Start a new page*** **Marks**

(a) Find the value of $\displaystyle\int_{-2}^{2} e^{2x}\,dx$ **2**

(b) Differentiate e^{-x^2}. Hence show that **4**

$$\int_{0}^{1} xe^{-x^2}\,dx = \frac{1}{2}\left(1-\frac{1}{e}\right)$$

(c) The number of bacteria, N, in a colony after t minutes grows according to the law $\dfrac{dN}{dt} = kN$, where k is constant.

(i) Show that $N = N_0 e^{kt}$, where N_0 is constant, is a solution of $\dfrac{dN}{dt} = kN$. **1**

(ii) If the number of bacteria is doubled in 200 minutes, find the value of k. **1**

(iii) How long, to the nearest minute, does it take for the number of bacteria to grow to 10 times the original number? **2**

(d) Evaluate $\displaystyle\int_{-1}^{1} x^2 e^x\,dx$ by using Simpson's rule with three function values. Answer correct to 2 significant figures. **3**

End of Examination

Q1

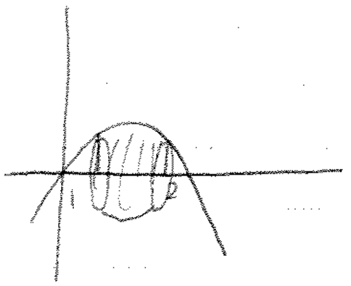
(a) $\frac{1}{33}(3x-2)^{11}+C$

(b) $y = x^2 - 3x + C$

$x = 1,\ y = 5 \quad \therefore C = 7$

$y = x^2 - 3x + 7$

(c)



$$V = \pi\int_1^2 (3x - x^2)^2\,dx$$

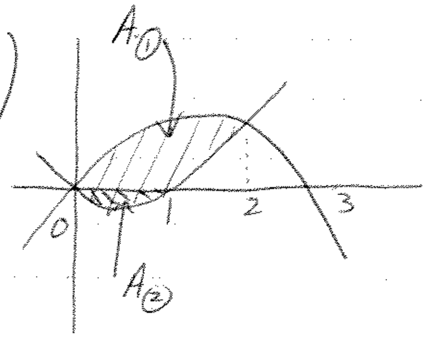
$$= \pi\int_1^2 (9x^2 - 6x^3 + x^4)\,dx$$

$$= \left[3x^3 - \frac{3}{2}x^4 + \frac{1}{5}x^5\right]_1^2$$

$$= 3.2^3 - \frac{3}{2}.2^4 + \frac{1}{5}.2^5 - \left(3 - \frac{3}{2} + \frac{1}{5}\right)$$

$$= 4\cdot 7 \text{ units}^3$$

(d)



$3x - x^2 = x^2 - x$

$2x(x-2) = 0$

$x = 0 \quad 2$

$$A_{①} = \int_0^2 (3x - x^2)\,dx - \int_1^2 (x^2 - x)\,dx$$

$$= \left[\frac{3}{2}x^2 - \frac{x^3}{3}\right]_0^2 - \left[\frac{1}{3}x^3 - \frac{x^2}{2}\right]_1^2$$

Q1 ctd

$A_{①} = \left[\frac{3}{2} \cdot 2^2 - \frac{2^3}{3} - 0\right] - \left[\frac{1}{3} \cdot 2^3 - \frac{2^2}{2} - \left(\frac{1}{3} - \frac{1}{2}\right)\right]$

$= 2\frac{1}{2}$

$A_{②} = \left|\int_0^1 (x^2 - x)\,dx\right|$

$= \int_1^0 (x^2 - x)\,dx$

$= \left[\frac{1}{3}x^3 - \frac{x^2}{2}\right]_1^0$

$= 0 - \left(\frac{1}{3} - \frac{1}{2}\right)$

$= \frac{1}{6}$

$A_{①} : A_{②} = 2\frac{1}{2} : \frac{1}{6}$

$= 15 : 1$

Q2

(a) $\log_e(4-3x) = \log_e x^2$, $x > 0$ from original question

$4 - 3x = x^2$

$x^2 + 3x - 4 = 0$

$(x+3)(x-1) = 0$

$x = -3$ or 1 , $x > 0$

$\therefore x = 1$ is only solution

(b) (i) $\dfrac{-4x}{3-2x^2}$

(ii) $2\cancel{x} \cdot \dfrac{\cancel{3}}{\cancel{3x}} + 2\ln 3x$

$= 2(1 + \ln 3x)$

(c) $y = x\ln x$

(i) $y' = \ln x + 1$

(ii) $y' = 0$ when $x = \frac{1}{e}$, $y = -\frac{1}{e}$

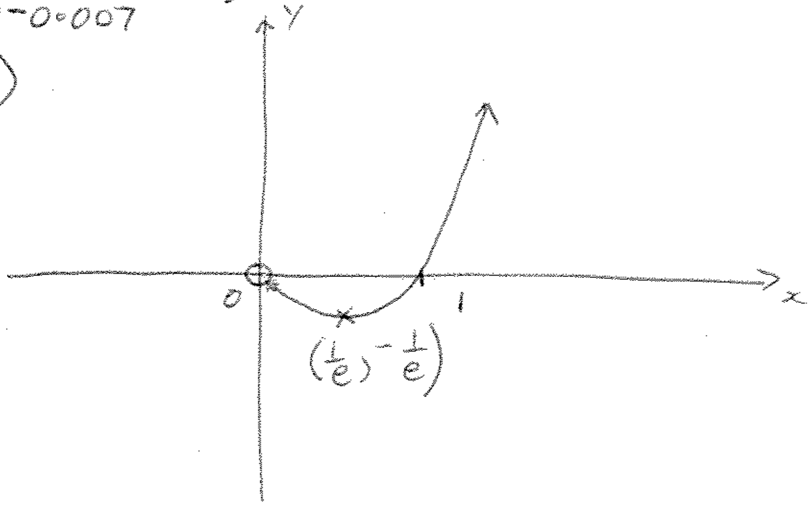
(iii) $y'' = \frac{1}{x}$

> 0 when $x = \frac{1}{e}$

$\therefore \left(\frac{1}{e}, -\frac{1}{e}\right)$ is a local minimum

(iv) -0.007

(v)



Q 3

(a) $\int_{-2}^{2} e^{2x}\,dx = \left[\frac{1}{2}e^{2x}\right]_{-2}^{2}$

$= \frac{1}{2}\left(e^{4} - e^{-4}\right)$

(b) $\frac{d}{dx}\left(e^{-x^2}\right) = -2xe^{-x^2}$

$\therefore \int_{0}^{1} -2xe^{-x^2}\,dx = \left[e^{-x^2}\right]_{0}^{1}$

$\int_{0}^{1} xe^{-x^2}\,dx = -\frac{1}{2}\left[e^{-x^2}\right]_{0}^{1}$

$= -\frac{1}{2}\left(e^{-1} - e^{0}\right)$

$= -\frac{1}{2}\left(\frac{1}{e} - 1\right)$

$= \frac{1}{2}\left(1 - \frac{1}{e}\right)$

(c) (i) $N = N_0 e^{kt}$

$\frac{dN}{dt} = k \times N_0 e^{kt}$

$= kN$

(ii) $2\cancel{N_0} = \cancel{N_0} e^{200k}$, $200k = \log 2$

$k = \frac{\log 2}{200}$

(iii) $10\cancel{N_0} = \cancel{N_0} e^{\frac{t}{200}\log 2}$

$\frac{t}{200}\log 2 = \log 10$

$t = \frac{200 \log 10}{\log 2}$

$= \cancel{2185}$ 664 min (nearest minute)

(d) $\int_{-1}^{1} x^2 e^x \, dx \doteqdot \frac{1}{3}\left[y_0 + y_2 + 4y_1\right]$

$= \frac{1}{3}\left(e^{-1} + e + 4e^0\right)$

$= \frac{1}{3}\left(e^{-1} + e + 2\right)$

$\approx 1 \cdot 7$